SAN JACINTO COLLEGE

STUDENT SUCCESS CENTER

Logarithms

Definition

A logarithmic equation $log_b(arg) = p$ is equivalent to the exponential equation $b^p = arg$, where b is the **base**, p is the **power**, and arg is the **argument**. Every logarithmic equation can be written as an exponential equation, and vice versa, just by properly rearranging the parts of the equations. The base must be nonzero, and the argument must be positive. (*Why?)*

For example, "What is the logarithm base 10 of 1000?" ($log_{10}(1000) = x$) is the *same question* as "What power does 10 need to be exponentiated to in order to equal 1000?" ($10^x = 1000$.) The answer to both questions is 3.

Special Cases

There are two particularly special cases:
$log_b(1) = 0$ for any base—because$b^0 = 1$for any base.
$log_b(b) = 1$ for any base—because$b^1 = b$for any base.
These are actually both specific cases of:
$log_b(b^n) = n$ for any base and any value of$log_b(b^n) = n$. As you might see, exponentiating and taking the logarithm "cancel"; the two functions are inverse. (This is true the other way around—$b^{log_b(n)} = n$.)

Other Useful Properties

The usefulness comes from the fact that logarithms reduce exponentiation to multiplication, and multiplication/division to addition/subtraction. These are highlighted in the following (base hidden for clarity, but these are true for any base):
$log(xy) = log(x) + log(y)$ **(product property)**
$log\left(\frac{x}{y}\right) = log(x) - log(y)$ **(quotient property)**
$log(x^n) = n \times log(x)$ **(power property)**

Using these properties to lengthen logarithmic expressions into more (yet simpler) terms is known as **expanding** logarithms, and using these properties to shorten logarithmic expressions into fewer (yet more complex) terms is known as **condensing** them. These three properties and the others above are all you need.

To see how all these properties might be useful, we look at the following:

Question: Evaluate $log_5(25)$.
Answer: $log_5(25) = log_5(5^2) = 2log_5(5) = 2$ (It is possible to skip the third step.)
Alternatively, solve $5^x = 25$.

Question: Evaluate $log_{20}(4) + log_{20}(5)$.
Answer: $log_{20}(4) + log_{20}(5) = log_{20}(4 \times 5) = log_{20}(20) = 1$

Question: Expand $log_6\left(\frac{x^3y^{1/2}}{z}\right)$.
Answer: $log_6\left(\frac{x^3y^{1/2}}{z}\right) = log_6(x^3y^{1/2}) - log_6(z) = log_6(x^3) + log_6(y^{1/2}) - log_6(z) = 3log_6(x) + \frac{1}{2}log_6(y) - log_6(z)$

Example Problems

1. $log_b \frac{x^2y}{z^2}$
2. $3lnx + 5lny - 6lnz$
3. $5e^x = 23$
4. $lne^x = ln21$
5. $5^{2x} = 7^{x+1}$
6. $\underset{log_4}{(3x + 2)} = 3$

P = $1523e^{0.02t}$ (For problems 7 and 8)

7. Find P when t=5
8. Find t when P=2100

Answers to Example Problems

1. $2log_bx + log_by - 2log_bz$
2. $ln\frac{x^3y^5}{z^6}$
3. $ln\left(\frac{23}{5}\right)$
4. $ln(21)$
5. $x = \frac{ln7}{2ln5-ln7}$
6. $x = \frac{62}{5}$
7. $P = 1683.175$
8. $t = 16.063$