SAN JACINTO COLLEGE

STUDENT SUCCESS CENTER

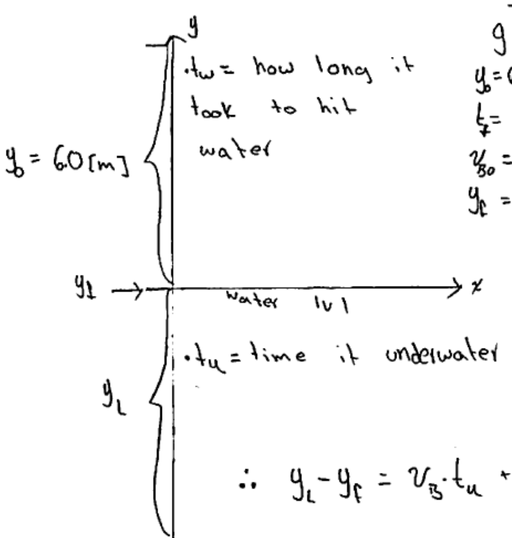
Physics – Kinematics, Projectile Motion, Free-Body Diagrams, and Rotational Motion

Kinematics and Projectile Motion Problem Solving Steps

1. Read and **Re-Read** the whole problem carefully before trying to solve it.
2. What is the question asking for? What specific **object** is moving and in what **time interval**? You can often choose the initial time to be $t=0$.
3. **Draw** a **Diagram** of the object with coordinate axes wherever applicable. Write down all the given information about the **object** of interest. Take note of what you want to know. You can orientate the xy **coordinate system** any way you choose.
4. Decide which **principle of physics** the object is experiencing.
5. Pick **equation** that solves for your unknown. Before using them, be sure they apply to your problem. Solve using algebra techniques. If the units do not match an error was made. Great way to check yourself.
6. Carry out numerical **calculations**, only round off in the final answer to the correct number of significant figures. When carrying out computation keep track of **units**. An equal sign implies the units on each side must be the same.
7. Are your results reasonable within the context of the problem?

Kinematics Example Problem

A lead ball is dropped into a lake from a diving board 6.0 [m] above the water. After entering the water, it sinks to the bottom with a constant velocity equal to the velocity with which it hit the water. The ball reaches the bottom 5.0 [s] after it is released. How deep is the lake?



Knowns	Unknowns
g	y_L
$y_0 = 6.0\,[m]$	t_w
$t_T = 5\,[s]$	v_B
$v_{B0} = 0\,[\frac{m}{s}]$	t_u
$y_f = 0\,[m]$	

$$y_f - y_0 = v_{B0} \cdot t_w + \frac{1}{2} g t_w^2$$

$$-6[m] = -\frac{1}{2}\left(9.81\left[\frac{m}{s^2}\right]\right) \cdot t_w^2$$

$$t_w = \sqrt{\frac{-6\,[m]}{-\frac{9.81}{2}\left[\frac{m}{s^2}\right]}} = 1.106\,[s]$$

$$t_T = t_w + t_u \qquad \therefore t_u = 3.894\,[s]$$

$$5[s] = 1.106[s] + t_u$$

→The lake is around 31.6 [m] deep.

$$v_B = \sqrt{v_{B0}^2 + 2g(y_f - y_0)}$$

$$v_B = \sqrt{2\left(9.81\left[\frac{m}{s^2}\right]\right)(-6[m])}$$

$$v_B = 11\left[\frac{m}{s}\right]$$

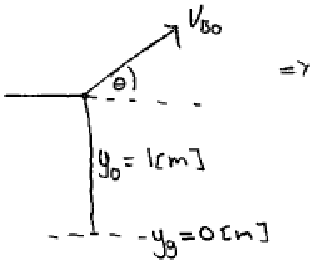
$$\therefore\ y_L - y_f = v_B \cdot t_u + \frac{1}{2} g t_u^2$$

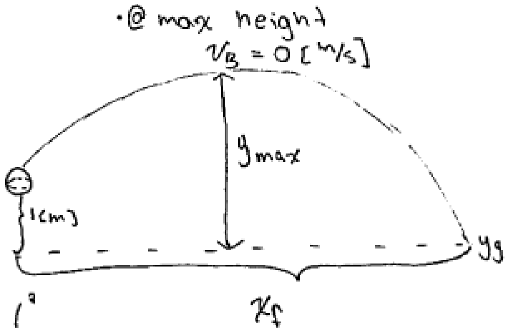
$$y_L = 11\left[\frac{m}{s}\right] \cdot 3.894[s] + \frac{1}{2}\left(-9.81\left[\frac{m}{s^2}\right]\right) \cdot (3.894[s])^2$$

$$y_L = -31.5416\,[m]$$

Projectile Motion Example Problem

A baseball is hit 1 [m] above the ground at an angle of 40˚ above the horizontal with an initial speed of 55 [m/s]. (A) How high will it reach in the sky? (B) How much time will it take to reach the ground? (C) How far will it travel before hitting the ground?





Knowns
$v_{Bo} = 55$ [m/s]
g $\theta = 40^\circ$
$y_0 = 1$ [m]

Unknowns
y_{max}
t_g
x_f

- Break up into Components

$v_{oy} = 55\sin(40^\circ)$ [m/s]

$v_{ox} = 55\cos(40^\circ)$ [m/s]

$\therefore v_B = v_{oy} + g\,t_{max}$

$0 = 55\sin(40^\circ)[m/s] + g\,t_{max}$

$\rightarrow t_{max} = 3.6038$ [s]

Ⓐ $y_{max} = y_0 + v_{oy}\cdot t_{max} + \frac{1}{2} g \cdot t_{max}^2$

$y_{max} = 1[m] + 55\cdot\sin(40^\circ)[\frac{m}{s}]\cdot 3.6038[s] - \frac{9.81[\frac{m}{s^2}]}{2}\cdot(3.6038[s])$

$y_{max} = 63$ [m]

Ⓑ $y_g - y_0 = v_{oy}\cdot t_g + \frac{1}{2} g \cdot t_g^2$ } • trajectory of ball is perabolic ∴ can use quadratic formula

$\rightarrow \frac{1}{2}(9.81[\frac{m}{s^2}])\cdot t_g^2 - 55\cdot\sin(40^\circ)[\frac{m}{s}]\cdot t_g - 1[m] = 0$

$$t_g = \frac{55\sin(40^\circ)[\frac{m}{s}] \pm \sqrt{(55\sin(40^\circ)[\frac{m}{s}])^2 - 4(\frac{9.81[\frac{m}{s^2}]}{2})(-1[m])}}{2(\frac{9.81[\frac{m}{s^2}]}{2})}$$

$\therefore t_g = 7.23578$ [s]

Ⓒ $v_{ox} = \frac{x_f - x_0}{t_g}$ $x_f = v_{ox}\cdot t_g$

$x_f = 55\cdot\cos(40^\circ)[\frac{m}{s}]\cdot 7.23578$ [s]

$\therefore x_f = 304.861$ [m]

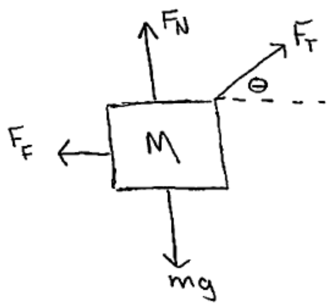
→ The max. height of the ball is 63 [m]. It would take around 7 [s] to fall to ground. Its horizontal distance would be around 305 [m]

Free-Body Diagrams and Newtonian Physics

1. **Draw a sketch** of the situation.
2. Draw a **free-body diagram** for the object of interest, showing all the forces acting on the object. Also, include any unknown forces that you must solve for. **Do not show any forces that chosen object exerts on other objects**. Instead draw free-body diagrams for each object separately, showing all the forces acting on that object. Only forces acting on a given object can be included in $\Sigma F = m$a equation for that object.
3. Choose xy axes for the coordinate system that simplifies the calculations. It often saves work if you choose one coordinate axis to be in the direction of the acceleration.
4. Break up $\Sigma F = m$a equation into its x and y components. (EX. $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$)
5. Solve the equation or equations for the unknowns.

Free-Body Diagram Example Problem 1

A wooden crate with a mass of 800 [kg] is pulled 25 [m] across a concrete floor by a man holing a rope 32˚ above the horizontal. If the tension in the rope is 160 [N] and the coefficient of friction between the crate and the floor is .55, what is the net force on the crate and the net work done on the crate?



Knowns

$m = 800[kg]$

$x = 25[m]$

$\theta = 32^\circ$

$F_T = 160[N]$

$\mu = .55$

Unknown

F_{NET}

W_{NET}

• F_{NET} is the sum of all the forces in a given direction

$\Sigma F_y = 0$

$F_N - F_T \sin(32^\circ) - mg = 0$

$F_N = mg + F_T \sin(32^\circ)$

$\therefore F_N = 7763[N]$

$F_f = \mu \cdot F_N$

$F_f = (.55)(7763[N])$

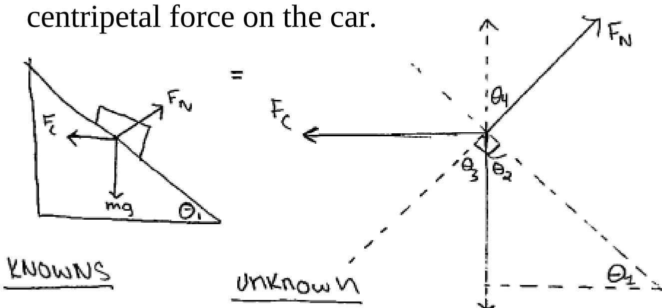
$\rightarrow \underbrace{F_{NET}}_{x\text{-direction}} = 160[N]\cos(32^\circ) - (.55)(7763[N])$

$F_{NET_x} = -4130[N]$

$W_{NET} = F_{NET} \cdot \Delta x = (-4130[N])(25[m]) = -103,351[J]$

Free-Body Diagram Example Problem 2

A car with a mass of 1050 [kg] travels around a curve of radius 300 [m] banked at a 14˚ angle. Find the maximum speed the car can take this curve without assistance from friction. Find the centripetal force on the car.



$\theta_2 = 90^\circ - \theta_1$

$\theta_2 = 76^\circ$

θ_2 & θ_3 are complementary

$\therefore \theta_3 = \theta_1$

$\theta_3 = \theta_4 \quad \therefore \theta_1 = \theta_4$

$\rightarrow F_{N_y} = F_N \cdot \cos\theta$

$F_{N_x} = F_N \cdot \sin\theta$

• THE CONDITION FOR THIS PROBLEM IS THAT $F_{N_x} = F_c$

KNOWNS

$m = 1050[kg]$

$r = 300[m]$

$\theta_1 = 14^\circ$

Unknown

v_c

F_c

$\Sigma F_x = 0$

$F_{N_x} - F_c = 0$

$F_N \sin\theta = \frac{m v_c^2}{r}$

$\Sigma F_y = 0$

$F_{N_y} - mg = 0$

$F_N \cos\theta_1 = mg$

$F_N = \frac{mg}{\cos\theta_1}$

$\therefore \frac{mg\sin\theta_1}{\cos\theta_1} = \frac{m v_c^2}{r}$

$\rightarrow v_c^2 = r \cdot g \cdot \tan\theta_1$

$v_c = \sqrt{(300[m])(9.8[\frac{m}{s^2}])\tan(14^\circ)}$

$v_c = 27.1[m/s]$

$\therefore F_c = \frac{m v_c^2}{r}$

$F_c = \frac{1050[kg] \cdot (27.1[\frac{m}{s}])^2}{300[m]} = 2570[N]$

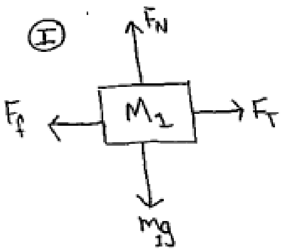
• THE MAX SPEED THE CAR CAN TAKE THE CURVE IS 27.1[m/s]

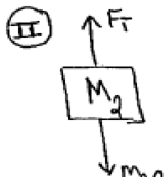
• THE CENTRIPETAL FORCE IS 2570[N]

Free-Body Diagram Example Problem 3

Bank robbers have pushed a 1000 [kg] safe to a second-story floor-to-ceiling window. They plan to break the window, then lower the safe 4.0 [m] to their truck. Not being too clever, they stack up 600 [kg] of furniture, tie the rope between the safe and the furniture, and place the rope over a pulley. Then they push safe out of the window. What is the safe's speed when it hits the truck? What is the force exerted on the truck by the safe? μ=.5

FREE BODY DIAGRAMS





KNOWNS

$M_1 = 600kg$ $\quad F_f = \mu \cdot F_N$

$M_2 = 1{,}000kg$

g $\quad \mu = .5$

$y_f = 4[m]$

Unknowns

V_s $\quad F_s$ $\quad F_N$ $\quad a$

• NOTE THAT M_2 IS NOT IN FREE FALL B/C OF F_T, WHICH IS ACTING UPWARDS ON IT ∴ M_1 & M_2 ACCELERATE @ SAME RATE $(a_y = a_x)$

• Sub one equation for another

ΣF_x $\qquad \Sigma F_y$

$F_T - F_f = m_1 a_x$ $\qquad F_N - m_1 g = 0$

$F_T - \mu \cdot F_N = m_1 a_x$ $\qquad F_N = m_1 g$

$\therefore F_T = m_1 a_x + \mu m_1 g$

(I) →

$m_1 a + \mu \cdot m_1 \cdot g = -m_2 a + m_2 g$ $\qquad a = \dfrac{g(m_2 - \mu m_1)}{m_1 + m_2}$

$a(m_1 + m_2) = g(m_2 - \mu m_1)$

$\therefore a = 4.29\ [m/s]$ $\qquad \rightarrow F_s = m_2 \cdot a$

ΣF_y

$F_T - m_2 g = -m_2 a_y$

$\therefore F_T = -m_2 a_y + m_2 g$

(II) →

$V_s^2 = V_{s0}^2 + 2a(y_f - y_0)$ $\qquad F_s = (1{,}000kg)(4.29\ [\frac{m}{s^2}])$

$V_s = \sqrt{2(4.29[\frac{m}{s^2}])(4.0[m])}$ $\qquad F_s = 4290\ [N]$

$V_s = 5.86\ [\frac{m}{s}]$

• The speed of the safe when it hits the truck is $5.86[m/s]$

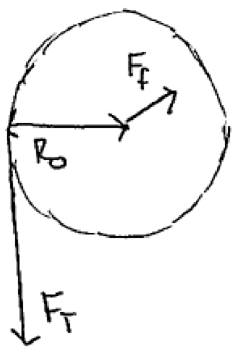
• The force the safe applies on truck is 4290 [N]

Rotational Motion

1. Draw a **diagram** of the object or objects that will be the system to be studied.
2. Draw a **Free-body diagram** for the object under consideration.
3. Identify the axis of rotation and determine the **torques** about it. Choose positive and negative directions of rotation, and assign the correct sign to each torque.
4. Apply **Newton's second law for rotation**, $\Sigma\tau = I\alpha$. If the moment of inertia is not given you need to solve for it first.
5. Also, apply **Newton's second law for translation,** $\Sigma F = m$a.
6. Solve.

Rotational Motion Example

A 15 [N] force is applied to a cord wrapped around a pulley of mass $M = 4.00$ [kg] and radius $R_0 = 33.0$ [cm]. The pulley accelerates uniformely from rest to an angular speed of 30.0 [rad/s] in 3.00 [s]. If there is a frictional torque $\tau_{fr} = 1.10$ [m. N] at the axle, determine the moment of inertia of the pulley. The pulley rotates about its center.



KNOWNS

$F_T = 15.0\,[N]$ $\quad \tau_f = 1.10\ m\cdot N$

$M = 4.00\,kg$

$R_o = 33.0\ cm$

$\omega = 30.0\ \frac{rad}{s}$

$t = 3.00\ [s]$

Unknown

I

$$\Sigma \tau = R_o F_T - \tau_{fr}$$

$$\Sigma \tau = (.330\,[m])(15.0\,[N]) - 1.10\ m\cdot N$$

$$\Sigma \tau = 3.85\ m\cdot N$$

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{30.0\ \frac{rad}{s}}{3.00\ [s]} = 10.0\ \frac{rad}{s^2}$$

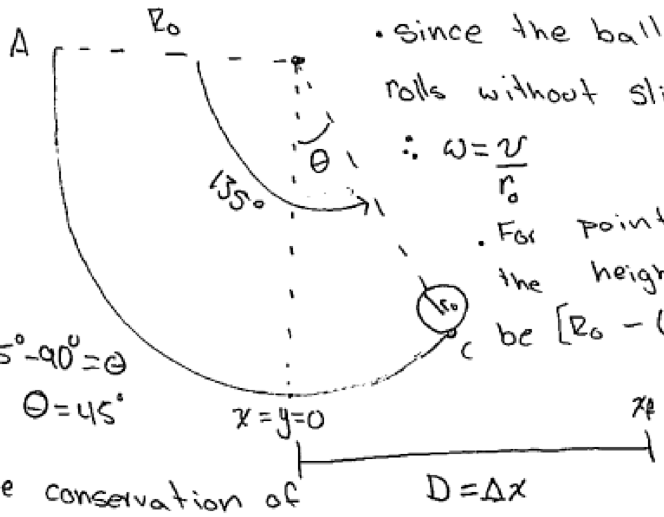
$$\Sigma \tau = I\alpha \quad \therefore \quad I = \frac{\Sigma\tau}{\alpha}$$

$$\rightarrow I = .385\ kg\cdot m^2$$

· Mg is ignored during calculations for $\Sigma\tau$ b/c we choose the center to be the axis of rotation ∴ zeros out

Rotational Plus Translational Motion Example

A small sphere of radios $r_0 = 1.5$ [cm] rolls without slipping on the track shown in Figure 1 whose radius is $R_0 = 26.0$ [cm]. The sphere starts rolling at a height R_0 above the bottom of the track. When it leaves the track after passing through an angle of 135° as shown, (A) what will be its speed, and (B) at what distance D from the base of the track will the sphere hit the ground?



$135° - 90° = \theta$

$\theta = 45°$

- Since the ball rolls without slipping

$\therefore \omega = \frac{v}{r_o}$

- For point C the height will be $[R_o - (R_o - R_o)\cos\theta]$

- Use conservation of mechanical energy @ point A to the energy @ point C

KNOWNS

$r_o = 1.5\,cm$

$R_o = 26.0\,cm$

g

Ⓐ $\rightarrow E_A = E_C$

$U_A = U_C + \underset{\text{linear movement}}{K_C} + \overset{\text{ROTATIONAL}}{K_C}$

$\therefore mgR_o = mg[R_o - (R_o - r_o)\cos\theta] + \frac{1}{2}mv_c^2 + \frac{1}{2}I\omega^2$

$\rightarrow mgR_o = mg[R_o - (R_o - r_o)\cos\theta] + \frac{1}{2}mv_c^2 + \frac{1}{2}\left(\frac{2}{5}mr_o^2\right)\frac{v_c^2}{r_o^2}$

$\therefore v_c = \sqrt{\frac{10}{7}g(R_o - r_o)\cos\theta} \approx 1.6\ [\frac{m}{s}]$

Ⓑ After leaving the track it becomes a projectile problem $\therefore$ break into components

$x_o = (R_o - r_o)\sin 45°$ $\quad y_o = R_o - (R_o - r_o)\cos 45°$

$v_{ox} = v_c \cos 45°$ $\quad v_{oy} = v_c \sin 45°$

FIND TIME IN Y-components

$y = y_o + v_{oy}t + \frac{1}{2}gt^2$

$\rightarrow R_o - (R_o - r_o)\cos 45° + v_c \sin 45°\, t - \frac{1}{2}gt^2 = 0$

$\rightarrow 4.90t^2 - 1.101t - .07178$

$\therefore t = .277\ [s]$

$\therefore v_{ox} = \frac{\Delta x}{t} = \frac{x_f - x_o}{t}$

$x_f = v_{ox} \cdot t + x_o$

$x_f = v_c \cos(45°) \cdot t + (R_o - r_o)\sin(45°)$

$x_f = D = .478\ [m]$